**Enrollment No: ______________________ Exam Seat No:______________________**

# C.U.SHAH UNIVERSITY

## Winter Examination-2015

**Subject Name :Advanced Real Analysis**

**Subject Code :** 5SC03ARC1 **Branch : M.Sc. (Mathematics)**

**Semester :3 Date :01/12/2015 Time :2.30 To 5.30 Marks : 70**

**Instructions:**

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

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## SECTION – I

**Q-1** **Attempt the Following questions .** **(07)**

**a.** What is difference between finite and σ-finite measure space? **(02)**

**b.** What is measurable function? **(02)**
**c.** What is $L^P$ Space? **(02)**
**d.** Define sign measure. **(01)**

**Q-2** **Attempt all questions** **(14)**
State and Prove monoton convergent theorem . **(07)**
State and prove beppo-levis's theorem . **(05)**
State Fatou's lemma. **(02)**

**OR**

**Q-2** **Attempt all questions** **(14)**
State and Prove Lebsegue dominated convergent theorem ( LDCT). **(07)**
Suppose f and g are integrable on E $\epsilon$ A then show that and **(05)**

(1) $\int_E (f+g)d\mu.=\int_E fd\mu+\int_E gd\mu$ (2) $\int_E \alpha f d\mu = \alpha\int_E fd\mu$
State bounded convergent theorem (BCT). **(02)**

**Q-3** **Attempt all questions** **(14)**
State and Prove Hahn-Decomposition theorem. **(07)**
State and prove $\ddot{H}$olders inequality . **(07)**

**OR**

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| **Q-3** | | Jordan –Decomposition theorem. | **(07)** |
| | | State and prove Minkowski's inequality. | **(07)** |

## SECTION – II

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| **Q-4** | | **Attempt the Following questions.** | **(07)** |
| | **a.** | State Caratheodary's Extension theorem. | **(02)** |
| | **b.** | What is Radon-Nikodym derivative? | **(02)** |
| | **c.** | Define Regular outer measure. | **(02)** |
| | **d.** | What is positive set ? | **(01)** |
| **Q-5** | | **Attempt all questions** | **(14)** |
| | | State and Prove Riesz - Reprentation theorem. | **(07)** |
| | | Let $\mu$ be a σ-finite measure on an algebra A .suppose $\mu^*$ is the induced outer measure. then E is measurable ⇔ E can be expressed as proper difference A-B where A∈ $A_{\sigma\delta}$ and B is a set with $\mu^*$(B) = 0. | **(07)** |
| | | **OR** | |
| **Q-5** | | Stat.e and prove Riesz-fischer's theorem | **(07)** |
| | | Show ($L^{\infty}$(μ) , $\|.\|_{\infty}$) are normed space. | **(07)** |
| **Q-6** | | **Attempt all questions** | **(14)** |
| | | Let f be monotonically increasing and right continuous .Suppose (a, b] ⊆ $\cup_{i\geq 1}(a_i , b_i]$ then F(b) - F(a) $\leq \sum_i F(b_i)$ – F($a_i$) . | **(07)** |
| | | Suppose (X, A , μ ) is a finite-measure space then $L^{\infty}$(μ) ≤ $L^p$(μ). | **(07)** |
| | | **OR** | |
| **Q-6** | | **Attempt all Questions** | **(14)** |
| | | Show that $L^P$(μ) are normed linear space. | **(07)** |
| | | State and prove lusin's theorem. | **(07)** |